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Fertility Measures and Population Projections in Denmark

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ABSTRACT

Fertility and population estimation techniques are examined in this study. Numerous countries undergo significant demographic changes that could significantly impact future fertility and demographic growth rates. the expanding population and fertility rates of a specific nation are the main topic of this article's analysis of some demographic trends. In the future, statistical models must account for measurement error and uncertainty when making predictions to achieve the highest degree of accuracy for demographic research. Fertility, which we assess as "Fertility Measures," is one of the key elements influencing a place's natural population expansion. The world's level of fertility varies greatly depending on the country, culture, socioeconomic status, and age of the individual. Calculations of population predictions depict the population's future growth under specific assumptions about how it will change in the future, typically to fertility, migration, and mortality. For this analysis, Denmark, a northern European nation, is considered. By the recent dataset of Denmark's population of the 4th quarter by age and live births (on the first day of the quarter), we also estimate the population projections by using the year 2011 as a base year to forecast the population growth of 2025. We concluded that total fertility had increased in 2021, and we predict the population forecast in 2025.

Keywords: Fertility Measures, Population Projection, Growth Models, Demographic Trends, Estimation.

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1. Introduction

Human population characteristics are reflected in demographic data. Population growth slowed during the 1970s due to demographic factors like fertility, death, migration, and urbanization. Sociology, politics, and marketing all make great use of demographic data. Frequently used demographics include gender, colour, age, income, skills (such as the ability to work while traveling), mobility, education level, home ownership, employment status, and even geography. Gille, H. 1949 [8], discusses the change in demographic trends in North European countries. Statistical models must account for measurement error and uncertainty when developing predictions to ensure the most significant degree of accuracy for demographic research in the future. Numerous deterministic techniques have been applied to depict these variables accurately. Demographic analysis is widely used in business strategies to define the population connected to the business's geographic area. It is used to calculate the size of the workforce and how many workers are moving around.

Vital statistics is the primary source of demographic data. Statistics on significant occurrences in a person's life can be utilized to learn crucial information about the population. Vital events cover life-and-death occurrences, such as births, deaths, marriages, and divorces. There are various means of obtaining vital statistics: Civil registration records document important events, such as births and deaths; census data with specific retrospective questions on fertility and mortality also reveal historical population patterns; health data can also help determine a person's health state and managing the risks connected to particular diseases. Statistics on actual events are typically gathered through a civil registration system, as this is the most reliable source for tracking details about deaths, births, and marriages. Indirect techniques of demographic estimation are then applied to these data sources to generate estimates that can be used for planning purposes.

In demography, fertility refers to the output or result of reproduction rather than the capacity to procreate. The probability of becoming pregnant depends on the pattern of sexual and pregnancy-preventative behaviours. The article by Bongaarts, J. 1982 [4], According to research, the majority of the variation in the population's fertility levels may be attributed to a relatively limited number of fertility-related factors. The percentage of married people, use of birth control, induced abortions, and postpartum fecundity are four variables. Demographic observed fertility or infertility results from a well-defined set of biological and behavioural characteristics, which buffer against the impact of culture, society, economic situations, living standards, and other related background factors on a person's reproductive behaviour. The characteristics of fertility are a group of these biological and behavioural traits.

Agriculture, less developed countries generally had lower fertility than industrialized, economically developed societies. Prioux, F. (2005) [19], said that in recent years, late fertility (defined as births to women who are 35 and older) has significantly increased in Western nations. The total age-specific fertility rates at 35 years or older are used to calculate late fertility. In addition, groups with higher incomes and higher education levels typically have lower fertility rates than groups with lower incomes and education levels. The end of fertility is now beginning to receive more attention from recent demographic and biological research on fertility at older ages. New demographic survey data on women's frequency of sexual activity indicate how exposed they are to the "risk" of becoming pregnant. Data from over a dozen and a half developing countries show that Pregnancy risk exposure for women declines with age.

However, the potential to increase elderly women's fertility may have significant ramifications in western cultures. Women of age groups for which childbearing has historically been biologically improbable and socially uncommon can become fertile. The social acceptability of extremely late childbirth will likely be positively impacted by extending older women's life expectancies, the

advantages of hormone replacement treatment, and overall improvements in older women's health and living situations in many western nations.

Population projections are calculations that depict the future growth of a population under specific assumptions regarding the direction of future population change, typically about migration, death, and fertility. Typically, they are solely formal computations that develop the ramifications of the given assumptions. Short-term projections are the norm even though the projection time is flexible because the margin of error that forecasts are susceptible to grows significantly as the projection period lengthens. Many of the techniques used for population forecasts can be used to estimate the population by size and composition at different points in the past and present. The population estimates from the most recent census are conducted annually based on the most recent census and subsequent years' vital statistics. Mondol, H., (2018) [17], discussed the population growth in Bangladesh. Methods of logistical and exponential growth are used to forecast Bangladesh's population increase. Northern Europe's only independent nation is the Kingdom of Denmark. Being the most southern of the Nordic countries, it only borders Germany to the south. Just over 5.8 million people live in Denmark. However, the population is anticipated to expand by over 100,000 people through 2026, partly due to the rise in average life expectancy in 2019. Denmark had a life expectancy at birth of almost 81 years. Since records have been kept, Denmark has experienced comparatively constant population growth. The population here had reached 797,584 in 1769, and by 1787, it had slightly increased to 841,806. During the 19th century, similar development persisted until Denmark's population reached 2,449,540. Therefore, statistics from Statistics Denmark should indicate that the population will finally surpass the six million level within the following ten years. Figure 1 shows the percentage of individuals of Danish heritage in the National and Regional Populations in 2021.

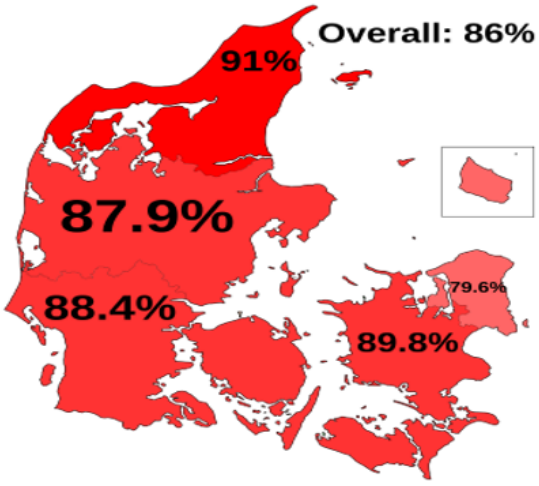

Figure 1: Population Percentages in 2021

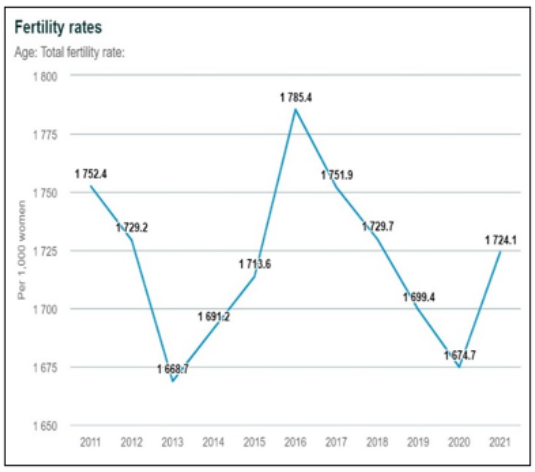

Figure 2: Total Fertility Rates (2011-2021)

2. Literature Survey:

According to the report by Becker, G. S. et al., (1990) [5], societies with prominent families favour big families and make little effort to support individual members. Societies with much human capital, on the other hand, act differently. As a result, there are two stable steady states: one with a lot of human and physical resources and small families. Bryant Robey et al., (1993) [20], describe how emerging countries are not going through the traditional demographic shift that occurred in many industrialized countries during the last century, despite what many observers had expected. Rindfuss, R. R. and Brewster, K. L. (1996) [21], This article focuses on the long-established connection between female labour force participation and fertility. The increasing rate of participation in the paid labour force by women lies at the heart of most discussions of fertility levels in the U.S. and other advanced industrial societies. In this article, we explore the implications of this variation for fertility behaviour. Wielandt and Knudsen (1997) [25], focuses on the Birth control experiences in Denmark. Since 1973, women in Denmark have had the option to obtain an induced abortion before the conclusion of the 12th week of pregnancy. The fact that many women seeking induced abortions are childless shows that the procedure can delay the first pregnancy effectively.

Rune Jacobsen et al., (1999) [11], analysed the genders of children born earlier in 363373 Danish families out of 613900 children on fertility rates to address the sex & combination preferences. the fertility rates were estimated using multiplicative Poisson Regression models. Jensen, T. K. et al., (2002) [12], discuss the declining fertility rates among Danish women between the ages of 15 and 19. The rise in induced abortion rates does not make up for the fall in fertility rates. In research from Sleebos, J. (2003) [22], Discuss, most OECD nations now have fertility rates significantly lower than required to ensure generational replacement. This article offers a comparative assessment of the data about this reduction's scope, timing, and character. Diprete, T. A., et al., (2003) [7], discuss the comparisons between country-specific costs of having children and parity-specific odds of having a subsequent birth. Empirical evidence supports the assumption that institutionally driven child costs influence the fertility patterns of industrialized countries. Alicia Adsera (2004) [1], highlights how low fertility, especially among younger women, results from Southern Europe's high unemployment rate and uncertain employment. Young women delay (or give up) having children to maximize their lifetime earnings and reduce their risk of unemployment. Public employment security and substantial maternity benefits increase fertility among women aged 25 to 29 and 30 to 34.

Gunnar Andersson (2004) [2], tells us that Swedish fertility has fluctuated somewhat throughout the entire period, whereas Danish and Norwegian fertility has gradually changed. In Norway and

Sweden, there has been a shift from declining to increasing childbearing rates. In the 1980s, Sweden experienced a change toward shorter birth intervals. Adsera, A. (2005) [3], Over the past 40 years, the average fertility rate in OECD countries has fallen considerably, from 2.9 in 1960 to 2.0 in 1975 to 1.6 in the late 1990s (reaching 1.25 in Southern Europe). Except for the United States, all industrialized countries have fertility rates well below the replacement rate of 2.1. Lutz, W. (2006) [15], This paper will review the arguments that either support the notion that European fertility rates are recovering or are continuing to drop. We will also talk about the compromises between immigration and fertility. Vahidnia, F. (2007) [23], studies with a 64% decrease in the total fertility rate between 1986 and 2000, Iran boasts one of the developing world's most influential family planning systems. The case study will aid researchers and politicians in Muslim nations in thinking about a successful family program as a practical idea that benefits health and human development.

Educational attainment had a positive influence on second birth rates for Danish women who has one child. As explained by the authors Mette Gerster., et al., (2007) [9], they examine that women with high education have a short time to have children in their desired number. Stefanie Brodmann., et al., (2007) [6], study second pregnancies, investigate how improved gender symmetry in childcare affects women's desire to have children, and concentrate primarily on the importance, with a significant focus on professionalism, of fatherly caring for women. Vos, A. E. (2009) [24], Discuss Europe has one of the world's lowest fertility rates (fewer than 2.1 children per woman). Declining fertility rates threaten economic development, and government finances must allow for increased pensions and health services. This article examines several factors influencing state, market, and family decisions. Ulla A. Hvidtfeldt et al., (2010) [10], Analysed the extent of changes in the period total fertility rate brought on by the timing of the births. First, the dataset “Danish fertility of women and couples (1980-2001)” based on certain factors is selected. This paper, described by Jain, A. K., & Ross, J.A. (2012) [13], discusses measures that are expected to increase female educational levels, reduce maternal mortality, and improve access to birth control, as well as the decline in fertility.

Marek Kupiszewski et al. (2016) [14], explore the effects of predicted changing demographics on many social areas, such as education, employment rate, health, etc. Based on demographic forecasts from 2005. Paine, R. R., et al., (2017) [18], explain that to estimate the population levels in the Copan Valley, Honduras, during the middle classic, people increased from the classic Maya settlement from A.D. 650–800 were employed. Population estimates depending on the rate of growth that characterizes late classic growth result in increases of between 40 and 100 percent. Anna Matysiak (2020) [16], explains that fertility was impacted during and before the great

recession in Europe from 2002 to 2014. Results show a significant relationship between the fertility decline and the increase in unemployment. The Authors viewed demographic trends, as well as the changes in attitudes and behaviours brought on by unification, from a broad sociological perspective.

3. Methodology

3.1 Notations:

P_o = The initial size of the population

P_t = Population at the time 't'

r_A = Intrinsic growth rate for Arithmetic Growth Model

r_G = Intrinsic growth rate for Geometric Growth Model

r_E = Intrinsic growth rate for Exponential Growth Model

3.2 Measures of Fertility:

The capacity to procreate, whether as a single person, a couple, a group, or a community, is referred to as fertility. The actual bearing of children is the intention. Some demographers favour using natality instead of fertility statistics. Population growth is a result of fertility. The average woman's reproductive years are 15 to 49. The key indicators of fertility included in 3.2.1 to 3.2.6.

3.2.1 Crude Birth Rate (CBR):

The number of births (children being born alive) every thousand in the midyear population is known as the crude birth rate.

$$CBR = \frac{Total\ Resident\ Live\ Births}{Total\ Population} \times 1000$$

Globally, CBR differs significantly from population to population. Population density is high in emerging nations and low in developed ones.

3.2.2 General Fertility Rate (GFR):

The general fertility rate is measured by the number of live births for every 1,000 fertile females aged 15 to 49 in a given year. Given that it considers the age and sex group capable of bearing children, the GFR is a more accurate indicator of fertility than the CBR (females 15-49 years of age).

$$GFR = \frac{Births\ in\ a\ stated\ period}{Mean\ number\ of\ women\ aged\ 15-49\ in\ the\ same\ period} \times 1000$$

The GFR is a more accurate metric for evaluating fertility between populations compared to CBR. About four times the CBR is the GFR.

3.2.3 Age-Specific Fertility Rate (ASFR):

The number of children born alive to females in a particular age group per 1000 females in that particular age group, for instance, (15-19), (20-24),…… (45-49) years of age, is known as the Age Specific Fertility Rate.

$$ASFR = \frac{Number\ of\ births\ to\ women\ in\ the\ age\ x\ to\ x+n}{Number\ of\ women\ aged\ x\ to\ x+n} \times 1000$$

This metric can be used to compare fertility patterns at various ages, assess the evolution of fertility at different ages, and compare fertility rates between multiple nations and people.

3.2.4 Total Fertility Rate (TFR):

The average number of infants a woman would have if she went through each stage of childbearing at the same rate is known as the total fertility rate. If age groups of five years are utilized, the total of the rates is multiplied by five. This metric provides a rough estimate of "finished family size."

$$\text{TFR} = \frac{\sum_{x=15-19}^{45-49} \text{ASFR}}{1000} \text{ X } 5$$

3.2.5 Gross Reproductive Rate (GRR):

The typical number of daughters a lady would have over her lifetime or childbearing age (15-49 years) if she remained in that age range is known as the gross reproduction rate. This rate is comparable to the TFR, but it only counts daughters and quantifies "reproduction" as a woman replicates herself through having a daughter. The GRR is computed by dividing the TFR by the percentage of female births (Sex Ratio at birth).

$$GRR = \frac{Number\ of\ Female\ Births}{Total\ Number\ of\ Births} \times TFR$$

3.2.6 Net Reproductive Rate (NRR):

The ratio of births to deaths determines population increase. Another indicator of population growth is NRR. It estimates how many female children would be born to a group of women who started out together and experienced current rates of fertility and mortality for the duration of their lives. The ratio of births to deaths influences population growth.

$$NRR = \frac{Births - Deaths}{Total\ Population} \times 100$$

Populations tend to grow if the net reproductive rate is more significant than one since more than one daughter will then replace each female. The population will often decrease if NRR is less than one. However, because it makes the false assumption that present fertility and motility will continue into the future, the net reproduction rate should not be used to predict the size of the future population.

3.3 Growth Models:

3.3.1 Arithmetic Growth Model:

A population expanding mathematically would grow by a fixed number of individuals per time. $P_t = P_o(1 + rt)$, Where, $r_A = \frac{1}{t}\left[\frac{P_t}{P_o} - 1\right]$

This assumption fails to hold when the change is minor and occurs over a short period of time. So, instead of a curve, arithmetic growth results in a linear trend in population growth.

3.3.2 Geometric Growth Model:

The geometric growth model makes the premise that the size of the population rapidly multiplies**.** This model predicts rapid population expansion**.** $P_t = P_o(1 + r)^t$, $r = antilog\left[\frac{1}{t} log_e \frac{P^t}{P_o}\right] - 1$

3.3.3 Exponential Growth Model:

If the growth of the population is compounded geometrically, then in a long range the population grows exponentially. It is expected that a population explosion will occur soon if the population increases exponentially. $P_t = P_O\, e^{rt}$, Where $r = \left[\frac{1}{t}\, ln \frac{P_t}{P_O}\right]$

4. Implementation:

For our analysis, we used the data of the fourth quarter population of 2021, which was acquired from the official website of the Danish census and arranged the information according to our needs. Table1shows the sorted dataset considered for implementation of the methodology.

Table 1: Population indices by Age, Births and deaths on the first day of the quarter

Age groups	Number of Births		Total Birth	Population		Total population	Deaths
	Male	Female		Male	Female		
Below 15	1	0	1	171570	162964	334534	22
15-19 years	116	104	220	175586	167890	343476	59
20-24 years	2240	2127	4367	192120	184753	376873	117
25-29 years	10794	10308	21102	206435	197730	404165	125
30-34 years	12444	11836	24280	192097	184079	376176	136
35-39 years	5686	5271	10957	168867	163968	332835	203
40-44 years	1189	1154	2343	174377	173248	347625	325
45-49 years	98	94	192	195796	196395	392191	638
50 & over	4	7	11	198497	196321	394818	1047

TOTAL	32572	30901	63473	1675345	1627348	3302693	2672

Total population of Denmark (2021): 5867412

Male Population: 2919210

Female Population: 2948202

4.1 Computation of Fertility Measures:

We obtained the fertility measures in table 2 by using the data in table 1

Table 2: Values obtained for Fertility Measures

Fertility Measures	Values Obtained
CBR	10.82
GFR	39.0039
ASFR (Total)	344.9563
TFR	1.7248
GRR	0.8397
NRR	1.04

4.2 Computation of Growth Models:

Using data from 2011 and 2021, we are estimating the population of 2025.

Total Population of Denmark (2011): 5579204

Total Population of Denmark (2021): 5867412

Table 3 displays equivalent outcomes for population projection**.**

Table 3: Intrinsic growth rates and Estimated population by using Growth models

Growth models	Intrinsic Growth Rate 'r.'	Estimated Population for 2025
Arithmetic Growth Model	0.0052	5989454
Geometric Growth Model	0.0051	5988026
Exponential Growth Model	0.0050	5985942

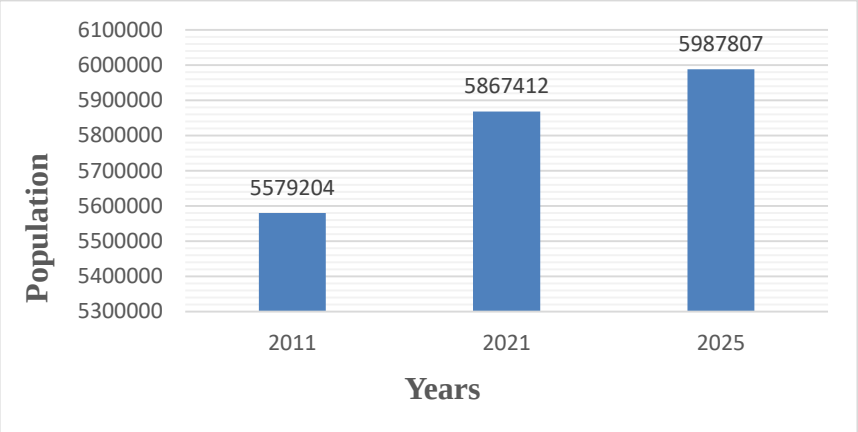

Figure 3: Population Growth between 2011 to 2025

Figure 3 depicts the Projected Population for 2025. It also shows the Growth of the Population from 2011 to 2025.

5. Conclusion

Demography mostly emphasizes the vital events of births, deaths, and migration. Fertility is the main factor responsible for population increase which can be quantified as fertility measures. The statistics shown in tables 1 and 2 are used to calculate the fundamental fertility measures for Denmark. The values attained are: Crude birth rates were 10.82. General fertility rates were 39.0039. Total fertility rates were 1.7248. Gross reproduction rates were 0.8397.Net reproduction rates were 1.04. Figure 1 shows the percentage of individuals of Danish heritage in the National and Regional Populations in 2021, and Figure 2 shows Denmark's Total Fertility Rates (2011-2021). There is an increase in their respective levels compared to the fertility rates in 2020. In comparison to fertility rates in 2020, the fertility rates in Denmark increased by 0.06%. Population projections using a mathematical model are one of the simplest methods to predict the future population using current demographics and situations. The effective application of population projection models then predicts the population of Denmark in 2025. Populations are affected by the use of mathematical, geometrical, and exponential models. The projected Populations of Denmark shown in table 3 were 5989454, 5988026, and 5985942. According to the predictions, the population of Denmark will expand by 2.07% when compared to the population in 2021, with intrinsic growth rates of 0.0051.

these findings for Denmark come from the core fertility measures. According to our research, fertility rates will increase, and population growth might be slower if trends in female educational attainment and access to contraception persist. In many nations, a persistent TFR below the replacement level would have negative economic, social, environmental, and geopolitical effects. In the coming years, it will be critical to developing policy choices that can accommodate the sustained low fertility while maintaining and improving female reproductive health.

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Dataset Reference

Denmark Census Data collected from the official website of Danish https://www.dst.dk/en